Real-time Polarization Calibration in the Phased-Array Beamformers for SKA1-Low and SKA1-Mid

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Abstract

This document discusses the technical challenges posed by the combined requirements to achieve high time resolution, perform automatic impulsive radio frequency interference (RFI) mitigation, and achieve 40 dB fidelity after polarimetric calibration. The proposed solution meets all of the requirements at minimal additional cost.

1 Introduction

The Level 1 system requirement specifications (revision 6B) relevant to this technical memo include:

- **SKA1-SYS_REQ-2473: RFI excision.** The SKA1 Telescopes shall automatically excise data that is corrupted by RFI.
- **SKA1-SYS_REQ-2961: SKA1_Mid Pulsar Timing resolution.** The timing resolution shall be better than 100 ns for SKA1_Mid Pulsar timing.
- **SKA1-SYS_REQ-2962: SKA1_Low Pulsar Timing resolution.** The timing resolution shall be better than 100 ns for SKA1_Low Pulsar timing.
- **SKA1-SYS_REQ-2965: SKA1_Mid Calibrated Polarisation Fidelity for Pulsars.** The SKA1_Mid telescope shall produce phase-resolved averages of the polarised flux from Pulsars with calibrated polarisation fidelity of at least 40 dB.
- **SKA1-SYS_REQ-2966: SKA1_Low Calibrated Polarisation Fidelity for Pulsars.** The SKA1_Low telescope shall produce phase-resolved averages of the polarised flux from Pulsars with calibrated polarisation fidelity of at least 40 dB.

The following sections outline the scientific rationale for the above requirements and the technical solutions that they necessitate.

1.1 Motivation for High-Fidelity Polarimetry

Table 1 of van Straten (2013) lists the systematic timing error induced by a 1% polarimetric calibration error for each of the 20 millisecond pulsars (MSPs) that have been regularly timed as part of the Parkes Pulsar Timing Array (PPTA) project. The most susceptible MSPs have systematic timing errors of the order of 300 nanoseconds. To achieve sensitivity to gravitational waves that induce arrival time fluctuations of the order of 10 nanoseconds over decades (and noting that Equation 4 of van Straten (2013) does not depend on b_k; therefore, to first order, arrival time distortion is linearly proportional to calibration error) it is desirable to reduce calibration error to 10^{-4}; i.e. achieve polarization fidelity of the order of 40 dB. A similar conclusion is reached in Section 4.3 of Cordes et al. (2004). The current Level 1 requirements specify that the intrinsic (i.e. antenna) cross-polarization ratio must be greater than 15 dB; therefore, further instrumental calibration is required.

1.2 Motivation for Phase Coherent Voltage Calibration

Ewan Barr has prepared a separate technical memo that justifies the 100 nanosecond maximum sampling interval (10 MHz minimum channel bandwidth) requirement for SKA1-Mid and challenges the need for similar time resolution on SKA1-Low. Therefore, this technical memo will focus on the implications of the requirement for a sampling rate of 10 MHz. Across each 10 MHz sub-band, the polarimetric response can be expected to vary on $\sim$ MHz scales (as observed at Parkes and GBT, for example). To calibrate the frequency response on scales smaller than the bandwidth, it is necessary to perform phase-coherent matrix convolution (van Straten, 2002). This can be done in either the time or frequency domain, as described below.

1.3 Motivation for Pre-Beam Formation RFI Mitigation

Some of the best pulsars for high-precision timing are very bright and exhibit pulse jitter Cordes & Downs (1985), also known as Stochastic Wideband Impulse Modulated Self-noise (SWIMS). The brightest pulses from these pulsars are easily confused with impulsive RFI; this is especially true after phased-array beam formation, where sensitivity to the pulsar signal is multiplied by the number of dishes. Flagging and clipping the brightest pulses will cause flux-dependent distortions to the mean pulse profile that translate into systematic timing error. Therefore, it is desirable to perform real-time impulsive RFI mitigation on the signal from each antenna (Mid Dish or Low Station), where sensitivity to the pulsar signal is dramatically reduced and the brightest pulses are less likely to be clipped. As a secondary consideration, some forms of terrestrial RFI will be local to a subset of SKA1 antennas, and flagging individual antenna before they enter the phased-array sum will reduce the total amount of data that are flagged and clipped and thereby improve S/N. Anecdotally, at Molonglo we find that we cannot properly phase up the array without first performing per-antenna RFI mitigation because the RFI signal dominates the cross-correlations.

2 Proposed Calibration Technique

Regardless of the calibration strategy chosen from the options presented in the following section, the high time resolution requirement (SKA1-SYS_REQ-2961) necessitates phase-coherent matrix convolution for polarimetric calibration, which can be performed in either the time or frequency domain. That is, when the instrumental response varies with frequency across the bandwidth of the signal, polarimetric calibration cannot be represented or performed as simple multiplication by a Jones matrix; rather, in the time domain, polarimetric transformations take the form of a vector convolution,

$$e'_x(t) = [j_{00} * e_x](t) + [j_{01} * e_y](t) \quad (1)$$

$$e'_y(t) = [j_{10} * e_x](t) + [j_{11} * e_y](t) \quad (2)$$

where the asterisks represent convolution, e_x and e_y are the input (complex-valued) electric field components and e'_x and e'_y are the output field components. The above system of equations can be written in vector notation as

$$\boldsymbol{e}'(t) = [\mathbf{J} \circledast \boldsymbol{e}](t) \quad (3)$$

Using the above notation, the output of a phased array can be written as

$$\boldsymbol{e}'(t) = \sum_i [h_i * \mathbf{J}_i \circledast \boldsymbol{e}_i](t) + \boldsymbol{n}_i(t) \quad (4)$$

where $h_i(t)$ is the (scalar) impulse response function of the delay filter applied to the i^{th} antenna in the beamformer, $\mathbf{J}_i(t)$ is the Jones matrix impulse response of the i^{th} antenna, $\boldsymbol{n}_i(t)$ is the noise in each dish, and

$$\boldsymbol{e}_i(t) = [g_i * \boldsymbol{e}](t) \quad (5)$$

is the electric field that arrives at the i^{th} antenna, which includes the (scalar) impulse response function of the geometric delay g_i of the i^{th} antenna with respect to the phase centre of the array. Apart from this geometric delay, an identical astronomical electric field vector $\boldsymbol{e}$ is received at each antenna, which is consistent with a point source (100% spatial coherence).

In a delay-calibrated phased array, the delay filter h_i cancels the geometric delay; i.e. $h_i(t) * g_i(t) = \delta(t)$, where δ is Dirac's delta function. The noise in each dish can be ignored because the off-pulse baseline is subtracted from the integrated pulse profile before further analysis is undertaken. Noting that scalar convolution is commutative - i.e. $h_i * \mathbf{J}_i = \mathbf{J}_i * h_i$ - and that both scalar and matrix convolution are distributive - i.e. $\mathbf{J}_i \circledast \boldsymbol{e} + \mathbf{J}_k \circledast \boldsymbol{e} = (\mathbf{J}_i + \mathbf{J}_k) \circledast \boldsymbol{e}$ - the output of a delay-calibrated phased array is

$$\boldsymbol{e}'(t) = \sum_i [\mathbf{J}_i \circledast \boldsymbol{e}](t) = \left[\left(\sum_i \mathbf{J}_i \right) \circledast \boldsymbol{e} \right](t) = [\mathbf{J} \circledast \boldsymbol{e}](t), \quad (6)$$

where

$$\mathbf{J}(t) = \sum_i \mathbf{J}_i(t). \quad (7)$$

That is, the response of the delay-calibrated array is simply the sum of the responses of the individual antenna.

Accordingly, subject to certain caveats, polarimetric calibration can be performed by correcting either

- the input signal from the i^{th} antenna using $\mathbf{J}_i(t)$ before summation; or
- the output signal from the beamformer using $\mathbf{J}(t)$ after summation.

The main caveat to the above conclusion is that the intrinsic cross-polarization ratio (IXR; e.g., see Foster et al., 2015, and references therein) of the sum should not exceed that of the individual elements. In principle, it is reasonable to expect that the phased-array sum will be better conditioned than the individual antennas; however, extreme counter-examples can also be contrived. For example, if half of the antennas in the array are rotated by 90 degrees, then the Jones matrix of the phased-array sum will be poorly conditioned (if not completely singular). These considerations could be used to specify new requirements that set a maximum error on the relative orientation of antenna elements.

2.1 Polarimetric Calibration and RFI Mitigation

Any RFI mitigation technique that masks individual antennas that are impacted by RFI before they are included in the voltage sum produced by the beamformer will produce (as an undesirable side effect) temporal variations in the polarimetric response of the phased-array beam. Therefore, it is necessary to either calibrate the input voltages from each antenna (before summation) or vary the response used to calibrate the output of the beamformer (after summation). Given an array of N antennas (dishes or stations) and M phased-array beams within the primary beam, there are four possible calibration methods that will produce a phased-array beam with a stable instrumental response:

- Full before summation $\mathcal{O}(NM)$: in the worst case, this would require real-time calibration of the dual-polarization voltage streams from all N antennas for all M phased-array beams.
- Boresight before summation $\mathcal{O}(N)$: calibrate only the instrumental response along the boresight of N antennas.
- Phased-array after summation $\mathcal{O}(M)$: calibrate the time-varying response of up to M phased-array beams.
- Hybrid $\mathcal{O}(N + M)$: calibrate only the instrumental response along the boresight of N antennas and then perform off-axis calibration of the time-varying response of M phased-array beams.

2.2 Considerations for SKA-Mid

For the purposes of pulsar timing on SKA1-Mid, either boresight before summation or phased-array after summation will produce acceptable results. The rationale for each method includes but may not be limited to the following arguments.

Boresight before summation: The density on the celestial sphere of pulsars that require high-fidelity polarimetric calibration is so low that it is unlikely that we will need to observe more than one within a primary beam. Calibration of N antennas before summation using a constant filter for each antenna is more computationally demanding (and therefore may consume more power) but may also be less computationally complex (and therefore require less engineering effort) than calibration with a time-varying response after summation.

Phased-array after summation: To account for the time variability of the instrumental response caused by RFI mitigation, it would be necessary to re-compute the sum of the Jones matrix impulse response every RFI mitigation interval and include only the responses of the dishes that were not masked. Given the 10 MHz sub-bands of SKA1-Mid, the Jones matrix impulse response could be recomputed every $\sim 10^4$ time samples or once per millisecond. Where $N_{\mathrm{R}FI}$ is the number of samples in each RFI mitigation interval and $N_{\mathbf{J}}$ is the number of taps in the Jones matrix impulse response of the phased array, a total of $N_{\mathrm{R}FI} + N_{\mathbf{J}}$ input samples must be processed for each $N_{\mathrm{R}FI}$ output samples, which introduces a small additional processing overhead.

2.3 Considerations for SKA1-Low

In the case of SKA1-Low, the band will be divided into 800 kHz wide channels by the station (LFAA) beamformer. We need more information (e.g. from MWA and/or LOFAR) to understand if the instrumental response varies significantly across 800 kHz. If not, then matrix convolution is not required and each 800 kHz channel can simply be multiplied by a single Jones matrix. Most likely, a hybrid solution would work best, in which each station is first calibrated for the boresight instrumental response before beam summation and then each beam is calibrated for the time-varying response post beam summation.

A Other Tradeoffs Considered

A.1 Fewer Beams = Higher Fidelity

For example, there could be two beam forming modes:

1. standard timing mode, in which the polarization fidelity requirements are significantly relaxed and either no calibration is performed in the beamformer or only a single Jones matrix is applied per antenna (i.e. neither matrix convolution in the time domain nor matrix multiplication in the frequency domain are performed); and
2. high-precision timing mode, in which the high polarization fidelity requirements are achieved.

A single high-precision timing beam could be formed at the cost of four standard timing beams in the beam former. The beamformer could then support up to

- 16 standard pulsars,
- 12 standard and 1 high-precision,
- 8 standard and 2 high-precision,
- 4 standard and 3 high-precision, or
- 4 high-precision pulsars.

A.2 Mask All or Mask None

A time variable Jones matrix is difficult to calibrate after adding antennas. However, if we assume that the fraction of time over which any antenna is masked is likely to be negligible then, without any great loss to S/N, it would be possible to replace each antenna mask with a single mask that is the logical AND of all antenna masks (assuming that zero means affected by RFI). That is, if any antenna is masked then all antennas are masked. Then the Jones matrix of the phased-array would be either constant or zero, which would be easier to calibrate after adding antennas.

B Outstanding Questions

1. What frequency resolution (or number of taps) is required to correct the instrumental response for Low and Mid Bands 1 through 5?
2. Instead of performing per-antenna RFI mitigation, should we focus effort on a smarter impulsive RFI excision algorithm that does not clip the brightest pulses from the pulsar?
3. What about matrix template matching (MTM; van Straten, 2006)? Can it be used to reduce the 40 dB requirement? Note that IXR is still important, as it determines how well the Jones matrices that describe the instrumental response can be inverted; MTM requires sufficiently high S/N to lock on to pulse phase and, because it's model includes more degrees of freedom, MTM requires higher S/N than standard scalar template matching methods (how much higher has not been quantified); and to achieve sufficient S/N, it is necessary to integrate over frequency. If the polarimetric response of the instrument varies with frequency, then integration over frequency will depolarize the signal, which cannot be corrected through calibration or MTM. Therefore, it is still necessary to correct the spectral variation of the instrumental response. Given time for further thought, analysis, and/or simulation we might be able to show that the spectral variations can be ignored; for example, to first order they might cancel.

References

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